

## Lehigh University Lehigh Preserve

---

Fritz Laboratory Reports

Civil and Environmental Engineering

---

1969

# On the limit analysis of stability of slope (Publication 69-21)

W. F. Chen

M. W. Giger .

H. Y. Fang

Follow this and additional works at: <http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports>

---

### Recommended Citation

Chen, W. F.; Giger, M. W. .; and Fang, H. Y., "On the limit analysis of stability of slope (Publication 69-21)" (1969). *Fritz Laboratory Reports*. Paper 396.

<http://preserve.lehigh.edu/engr-civil-environmental-fritz-lab-reports/396>

This Technical Report is brought to you for free and open access by the Civil and Environmental Engineering at Lehigh Preserve. It has been accepted for inclusion in Fritz Laboratory Reports by an authorized administrator of Lehigh Preserve. For more information, please contact [preserve@lehigh.edu](mailto:preserve@lehigh.edu).



Soil Mechanics, and Theories of Plasticity

# ON THE LIMIT ANALYSIS OF STABILITY OF SLOPES

by  
W. F. Chen  
M. W. Giger  
H. Y. Fang

M  
620.6  
L527F  
NO.355.4

UNIVERSITY OF ILLINOIS AT CHICAGO  
INSTITUTE OF ENGINEERING

Fritz Engineering Laboratory Report No. 355.4

Soil Mechanics, and Theories of Plasticity

ON THE LIMIT ANALYSIS OF STABILITY OF SLOPES

by

W. F. Chen  
M. W. Giger  
H. Y. Fang

This work has been carried out as part of an investigation sponsored by the Envirotronics Corporation.

Fritz Engineering Laboratory  
Department of Civil Engineering  
Lehigh University  
Bethlehem, Pennsylvania

March 1969

Fritz Engineering Laboratory Report No. 355.4

# On the Limit Analysis of Stability of Slopes

By

W. F. Chen\*, M. W. Giger\* and H. Y. Fang\*

## ABSTRACT

The upper bound theorem of the generalized theory of perfect plasticity is applied to obtain complete numerical solutions for the critical height of an embankment. A rotational discontinuity mechanism (logarithmic spiral) is assumed in the analysis. The analysis includes the existing solutions as a special case, so that it may be considered a generalization of previous solutions.

---

\* Assistant Professor of Civil Engineering, Research Assistant and Director of Geotechnical Engineering Division, respectively Fritz Engineering Laboratory, Lehigh University.

## TABLE OF CONTENTS

ABSTRACT	Page
1. INTRODUCTION.....	1
2. CRITICAL HEIGHT OF AN EMBANKMENT.....	4
3. NUMERICAL RESULTS.....	8
4. CONCLUSION.....	9
5. ACKNOWLEDGEMENTS.....	10
6. TABLES AND FIGURES.....	11
7. NOMENCLATURE.....	12
8. REFERENCES.....	13

## 1. INTRODUCTION

Application of limit theorems of the generalized theory of perfect plasticity (Drucker, Prager and Greenberg, 1952) to stability problems in soil mechanics has shown promising results. Typical examples include the stability of slopes (Drucker and Prager, 1952; Drucker, 1953), the bearing capacity of footings (Shield, 1954; 1955; Cox, Eason and Hopkins, 1961; Chen, 1969) and the earth pressure on retaining walls (Chen, 1968; Finn, 1968). A review of earlier applications of the theorems to stability problems in soil mechanics has been given by Finn (1968), Chen and Scawthorn (1968).

The approach is based on the assumption that a real soil will be deformed according to the flow rule associated with the Coulomb yield condition. The implications of this basic assumption are far reaching. When applied to stability problems in soil mechanics for which satisfactory solutions already exist, the approach often yields solutions which are in good agreement with the existing results obtained by limit equilibrium analysis (plastic equilibrium), (for example, see Terzaghi, 1943). Unfortunately, the volume expansion, which is predicted by the assumption to accompany the shearing action, is enormously in excess of that observed (Drucker, Gibson and Henkel, 1957).

More sophisticated theories on stress-strain relations for soils have been proposed by Drucker, Gibson and Henkel (1957), Jenike and Shield (1959), Spencer (1964) and Weidler and Paslay (1966) in an attempt to overcome some of the known deficiencies in the previous theories. While such improvements may be more

accurate than the assumption of perfect plasticity, they also require more complex analytical techniques for their solution.

On the other hand, the methods of limit equilibrium take no account of the soil deformation associated with the limiting state of stress, and hence have an unknown degree of inaccuracy when applied to problems in soil mechanics.

A compromise between these approaches is to consider the stress-strain relationship for soils in an idealized manner. This idealization, termed perfect plasticity, established the limit theorems on which limit analysis is based. As noted earlier, for stability problems in soil mechanics, there appears to be reasonable justification for the adoption of a limit analysis approach based on Coulomb's yield criterion and its associated flow rule.

The problem considered here is the critical height of an embankment with slope angles  $\alpha$  and  $\beta$  as shown in Fig. 1. The limit analysis of this problem for a vertical bank ( $\alpha=0^\circ$  and  $\beta=90^\circ$ ) is given by Drucker and Prager (1952). They note that the upper bound solution can be improved by considering a rotational discontinuity (logarithmic spiral) instead of the translation discontinuity (plane surface) used by them. In order to provide a more complete means of comparing the results obtained by the limit analysis approach with those obtained by the limit equilibrium approach, and also yield additional theoretical evidence as to the

validity and limitations of the theory of perfect plasticity as applied to stability problems in soil mechanics, it is of value, therefore, to obtain the more elaborate upper bound solution for the critical height of an embankment using a logarithmic spiral surface discontinuity mechanism. This is described in the present paper.



## 2. CRITICAL HEIGHT OF AN EMBANKMENT

The upper bound theorem of limit analysis states that the embankment shown in Fig. 1 will collapse under its own weight if, for any assumed failure mechanism, the rate of work done by the soil weight exceeds the internal rate of dissipation. Equating external and internal energies for any such mechanism thus gives an upper bound on the critical height.

A rotational discontinuity mechanism is shown in Fig. 1. The triangular-shaped region ABC rotates as a rigid body about the center of rotation O (as yet undefined) with the materials below the logarithmic surface BC remaining at rest. Thus, the surface BC is a surface of velocity discontinuity. The assumed mechanism can be specified completely by three variables. For the sake of convenience, one selects the slope angles  $\theta_o$  and  $\theta_h$  of the chords OB and OC, respectively and the height H of the embankment. From the geometrical relations it may be shown that the ratios,  $H/r_o$  and  $L/r_o$ , can be expressed in terms of the angles  $\theta_o$  and  $\theta_h$  in the forms

$$\frac{H}{r_o} = \frac{\sin \beta}{\sin(\beta-\alpha)} \left\{ \sin(\theta_h+\alpha) \exp[(\theta_h-\theta_o) \tan \phi] - \sin(\theta_o+\alpha) \right\} \quad (1)$$

and

$$\frac{L}{r_o} = \frac{\sin(\theta_h-\theta_o)}{\sin(\theta_h+\alpha)} - \frac{\sin(\theta_h+\beta)}{\sin(\theta_h+\alpha) \sin(\beta-\alpha)} \left\{ \exp[\theta_h-\theta_o) \tan \phi] \right. \\ \left. \sin(\theta_h+\alpha) - \sin(\theta_o+\alpha) \right\} \quad (2)$$

A direct integration of the rate of external work due to the soil weight in the region ABC is very complicated. An easier alternative is first to find the rates of work  $\dot{W}_1$ ,  $\dot{W}_2$ , and  $\dot{W}_3$  due to the soil weight in the regions OBC, OAB, and OAC, respectively. The rate of external work for the region ABC is then found by the simple algebraic summation,  $\dot{W}_1 - \dot{W}_2 - \dot{W}_3$ . It is found, after some simplification, that the rate at which work is done by the soil weight in the region ABC is

$$\gamma r_o^3 \Omega (f_1 - f_2 - f_3) \quad (3)$$

where  $\gamma$  is the unit weight of the soil,  $\Omega$  is the angular velocity of the region ABC, the functions  $f_1$ ,  $f_2$ , and  $f_3$  are defined as

$$f_1(\theta_h, \theta_o) = \frac{1}{3 (1 + 9 \tan^2 \phi)} \left\{ (3 \tan \phi \cos \theta_h + \sin \theta_h) \exp[3 (\theta_h - \theta_o) \tan \phi] - (3 \tan \phi \cos \theta_o + \sin \theta_o) \right\} \quad (4)$$

$$f_2(\theta_h, \theta_o) = \frac{1}{6} \frac{L}{r_o} (2 \cos \theta_o - \frac{L}{r_o} \cos \alpha) \sin(\theta_o + \alpha) \quad (5)$$

and  $L/r_o$  is a function of  $\theta_h$  and  $\theta_o$  (Eq. 2).

$$f_3(\theta_h, \theta_o) = \frac{1}{6} \exp[(\theta_h - \theta_o) \tan \phi] \left\{ [\sin(\theta_h - \theta_o) - \frac{L}{r_o} \sin(\theta_h + \alpha)] \left\{ \cos \theta_o - \frac{L}{r_o} \cos \alpha + \cos \theta_h \exp[(\theta_h - \theta_o) \tan \phi] \right\} \right\} \quad (6)$$

The internal dissipation of energy occurs along the discontinuity surface BC. The differential rate of dissipation of energy along the surface may be found by multiplying the differential area,  $r d\theta / \cos \phi$ , of this surface by the cohesion  $c$  times the tangential discontinuity in velocity,  $V \cos \phi$ , across the surface (Drucker and Prager, 1952). The total internal dissipation of energy is found by integration over the whole surface.

$$\int_{\theta_o}^{\theta_h} c (V \cos \phi) \frac{r d\theta}{\cos \phi} = \frac{c r_o^2 \Omega}{2 \tan \phi} \left\{ \exp[2(\theta_h - \theta_o) \tan \phi] - 1 \right\} \quad (7)$$

Equating the external rate of work, Eq. 3, to the rate of internal energy dissipation, Eq. 7, gives

$$H = \frac{c}{\gamma} f(\theta_h, \theta_o) \quad (8)$$

where  $f(\theta_h, \theta_o)$  is defined as

$$f(\theta_h, \theta_o) = \frac{\sin \beta \left\{ \exp[2(\theta_h - \theta_o) \tan \phi] - 1 \right\}}{2 \sin (\beta - \alpha) \tan \phi (f_1 - f_2 - f_3)} \quad (9)$$

$$\left\{ \sin (\theta_h + \alpha) \exp[(\theta_h - \theta_o) \tan \phi] - \sin (\theta_o + \alpha) \right\}$$

By the upper bound theorem of limit analysis, Eq. 8 gives an upper bound for the critical value of the height,  $H_c$ . The function  $f(\theta_h, \theta_o)$  has a minimum value when  $\theta_h$  and  $\theta_o$  satisfy

the conditions

$$\frac{\partial f}{\partial \theta_h} = 0 \quad \text{and} \quad \frac{\partial f}{\partial \theta_o} = 0 \quad (10)$$

Solving these equations and substituting the values of  $\theta_h$  and  $\theta_o$  thus obtained into Equation 8 yields a least upper bound for the critical height,  $H_c$ , of an inclined slope. Denoting  $N_s = \text{Min. } f(\theta_h, \theta_o)$ , one obtains

$$H_c \leq \frac{c}{\gamma} N_s \quad (11)$$

The dimensionless number  $N_s$  is known as the stability factor of the embankment. The value of  $N_s$  depends not only on the slope angles  $\alpha$  and  $\beta$  but also on the angle of internal friction,  $\phi$ .

### 3. NUMERICAL RESULTS

A complete solution to this problem has been obtained by numerical methods, the numerical work being performed on a CDC 6400 digital computer. The results of these computations are graphically represented in Figures 2, 3, and 4 for the special cases:  $\alpha = 0$  or  $\beta = 90^\circ$ , and tabulated numerically in Table 1 for the more general cases.

For the case  $\alpha = 0$ , the critical values of  $\theta_h$ ,  $\theta_o$  and  $L/r_o$  corresponding to Equation 10 are plotted against the inclined slope angle  $\beta$  for  $\phi = 20^\circ$ . The relation between the inclined slope angle  $\beta$  and the stability factor  $N_s$  for various values of  $\beta$  is shown in Fig. 3. It is found that the results obtained are practically identical to those results obtained by the  $\phi$ -circle method of Taylor (1937), and by the slice method of Fellenius (1927).

For the case  $\beta = 90^\circ$ , the relation between  $\alpha$  and  $N_s$  for various values of  $\phi$  is shown in Fig. 4. Table 1 gives the value of  $N_s$  for various combination of the slopes  $\alpha$  and  $\beta$ . Since there are no existing solutions available for the case where  $\alpha \neq 0$ , the present results will certainly provide useful information concerning the problem, although the answers may not be exact.

Table 2 presents a comparison of various limit equilibrium solutions with the upper bound limit analysis solution, for different inclined slopes  $\beta$  with the slope  $\alpha = 0$ . It may be seen that agreement is good.

#### 4. CONCLUSION

The upper bound theorem of limit analysis may be used to predict the critical height of an embankment. The general formulation of this problem is straight forward and the numerical results for the case  $\alpha = 0$  agree with already existing limit equilibrium solutions. Although no limit equilibrium results were available for the case of  $\alpha \neq 0$ , the limit analysis solutions derived are expected to be valid, and provide additional insight into the overall problem of embankment stability.

## 5. ACKNOWLEDGEMENTS

The work described in this paper was conducted in the Geotechnical Engineering Division, Fritz Engineering Laboratory, Lehigh University, as part of the research program on Soil Plasticity. This particular study is sponsored by the Envirotronics Corporation.

The authors extend their appreciation to Professor T. J. Hirst for his review of the manuscript, to Miss Beverly E. T. Billets for her help in typing the manuscript, and to Mr. Gera for preparing the drawings.

## 6. TABLES AND FIGURES



TABLE 1 STABILITY FACTOR  $N_s = H_c \frac{Y}{C}$   
BY LIMIT ANALYSIS

FRICTION ANGLE $\phi$ in degrees	SLOPE ANGLE $\alpha$ in degrees	SLOPE ANGLE $\beta$ IN DEGREES					
		90	75	60	45	30	15
0	0	3.83	4.57	5.25	5.86	6.51	7.35
5	0	4.19	5.14	6.17	7.33	9.17	14.80
	5	4.14	5.05	6.03	7.18	8.93	14.62
10	0	4.59	5.80	7.26	9.32	13.53	45.53
	5	4.53	5.72	7.14	9.14	13.26	45.15
	10	4.47	5.61	6.98	8.93	12.97	44.56
15	0	5.02	6.57	8.64	12.05	21.71	
	5	4.97	6.49	8.52	11.91	21.50	
	10	4.90	6.39	8.38	11.73	21.14	
	15	4.83	6.28	8.18	11.42	20.59	
20	0	5.51	7.48	10.39	16.18	41.27	
	5	5.46	7.40	10.30	16.04	41.06	
	10	5.40	7.31	10.15	15.87	40.73	
	15	5.33	7.20	9.98	15.59	40.16	
	20	5.24	7.04	9.78	15.17	39.19	
25	0	6.06	8.59	12.75	22.92	120.0	
	5	6.01	8.52	12.65	22.78	119.8	
	10	5.96	8.41	12.54	22.60	119.5	
	15	5.89	8.30	12.40	22.37	118.7	
	20	5.81	8.16	12.17	21.98	117.4	
	25	5.71	7.97	11.80	21.35	115.5	
30	0	6.69	9.96	16.11	35.63		
	5	6.63	9.87	16.00	35.44		
	10	6.58	9.79	15.87	35.25		
	15	6.53	9.67	15.69	34.99		
	20	6.44	9.54	15.48	34.64		
	25	6.34	9.37	15.21	34.12		
	30	6.22	9.15	14.81	33.08		
35	0	7.43	11.68	20.94	65.53		
	5	7.38	11.60	20.84	65.39		
	10	7.32	11.51	20.71	65.22		
	15	7.26	11.41	20.55	65.03		
	20	7.18	11.28	20.36	64.74		
	25	7.11	11.12	20.07	64.18		
	30	6.99	10.93	19.73	63.00		
	35	6.84	10.66	19.21	60.80		
40	0	8.30	14.00	28.99	185.6		
	5	8.26	13.94	28.84	185.5		
	10	8.21	13.85	28.69	185.3		
	15	8.15	13.72	28.54	185.0		
	20	8.06	13.57	28.39	184.6		
	25	7.98	13.42	28.16	184.0		
	30	7.87	13.21	27.88	183.2		
	35	7.76	12.95	27.49	182.3		
	40	7.61	12.63	26.91	181.1		

TABLE 2 COMPARISON OF STABILITY FACTOR  $N_s = H_c \frac{Y}{C}$   
 BY METHODS OF LIMIT EQUILIBRIUM AND LIMIT ANALYSIS  
 $\alpha = 0$

SLOPE ANGLE $\beta$ degrees	FRICTION ANGLE $\phi$ degrees	CURVED FAILURE SURFACE			
		LIMIT EQUILIBRIUM *			LIMIT ANALYSIS
		SLICES	$\phi$ CIRCLE	LOG- SPIRAL	LOG- SPIRAL
90	0	3.83	3.83	3.83	3.83
	5	4.19	4.19	4.19	4.19
	15	5.02	5.02		5.02
	25	6.06	6.06	6.06	6.06
75	0	4.57	4.57	4.57	4.57
	5	5.13	5.13		5.13
	15	6.49	6.52		6.57
	25	8.48	8.54		8.59
60	0	5.24	5.24	5.24	5.25
	5	6.06	6.18	6.18	6.17
	15	8.33	8.63	8.63	8.64
	25	12.20	12.65	12.82	12.75
45	0	5.88	5.88	5.88	5.86
	5	7.09	7.36		7.33
	15	11.77	12.04		12.05
	25	20.83	22.73		22.92
30	0	6.41	6.41	6.41	6.51
	5	8.77	9.09		9.17
	15	20.84	21.74		21.71
	25	83.34	111.1	125.0	120.0
15	0	6.90	6.90	6.90	7.36
	5	13.89	14.71	14.71	14.81
	10		43.62		45.53

\*Taylor, (1948)



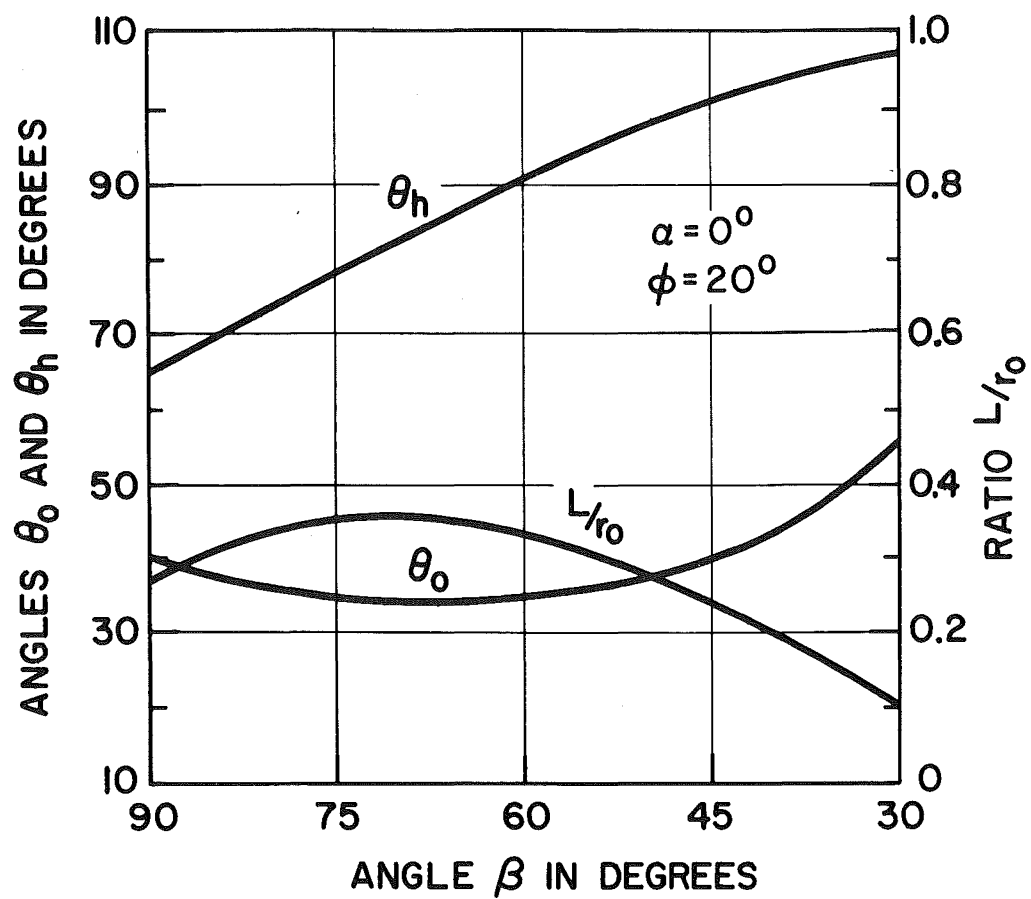


Fig. 2 CRITICAL VALUES FOR  $\theta_h$ ,  $\theta_o$  AND  $L/r_o$   
 AS A FUNCTION OF SLOPE ANGLE  $\beta$

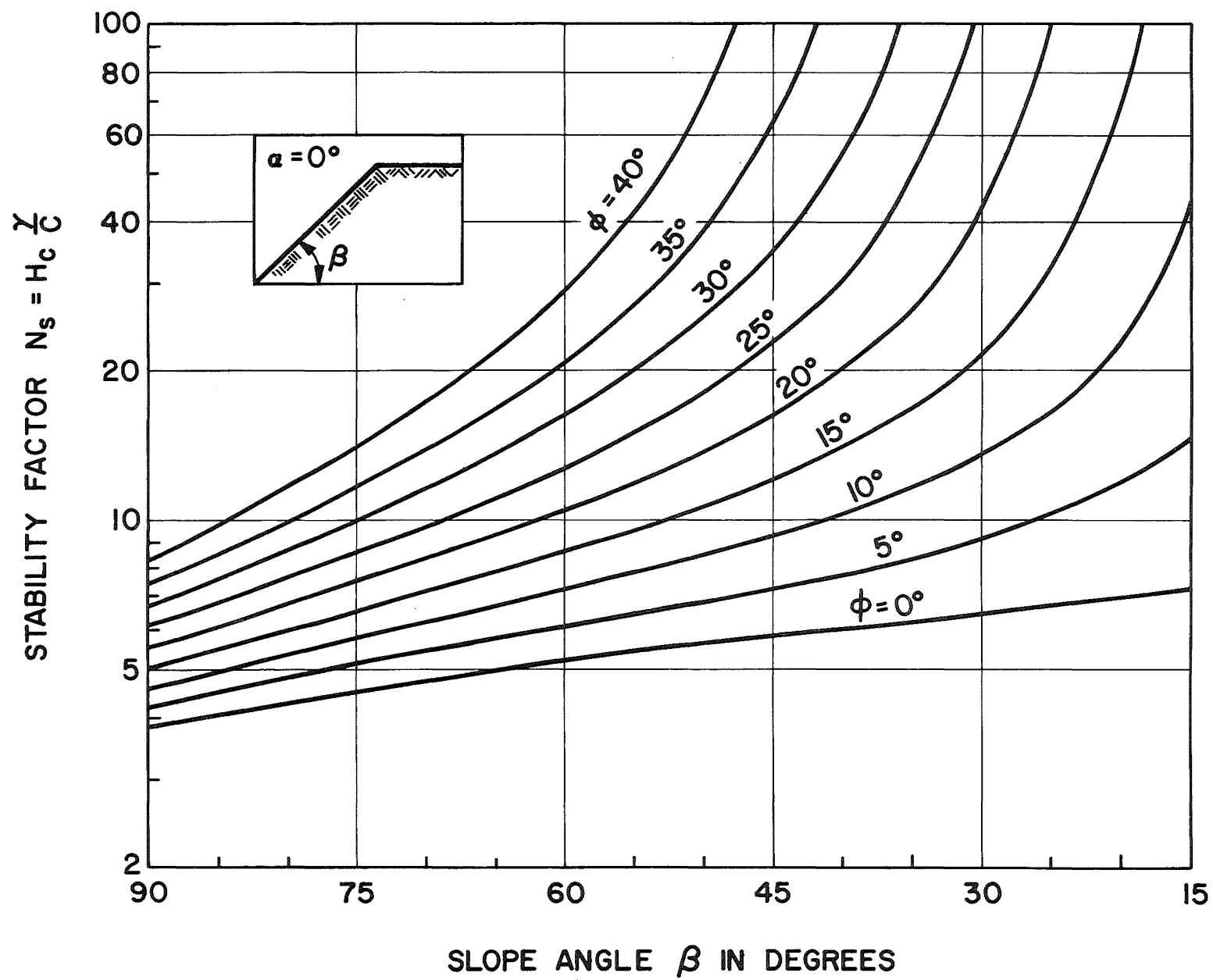


Fig. 3 STABILITY FACTORS  $N_s$ , AS A FUNCTION OF SLOPE ANGLE  $\beta$

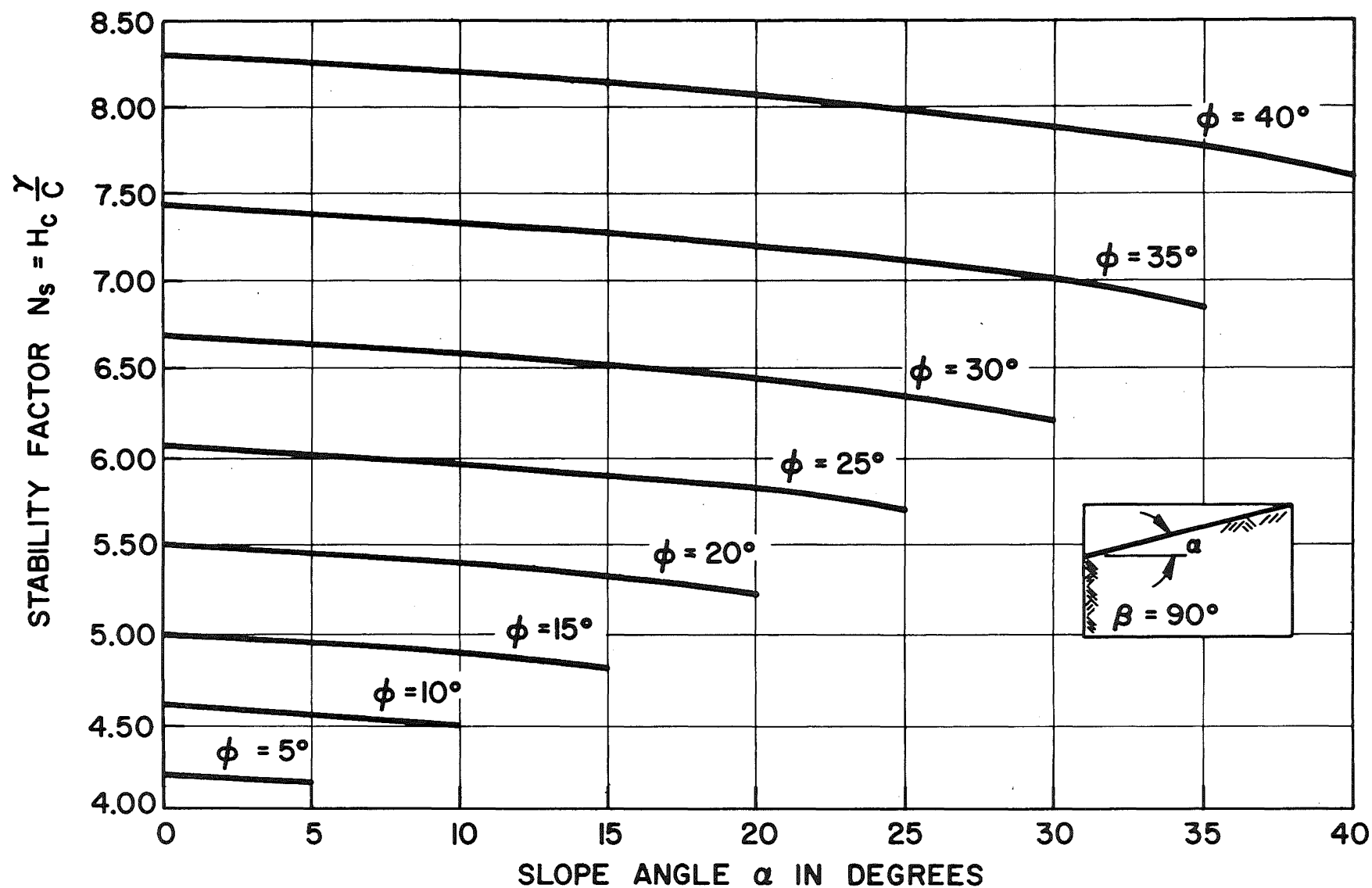


Fig. 4 STABILITY FACTORS  $N_s$ , AS A FUNCTION OF SLOPE ANGLE  $\alpha$

## 7. NOMENCLATURE

$c$	cohesion
$f_1$	function defined in Eq. (4)
$f_2$	function defined in Eq. (5)
$f_3$	function defined in Eq. (6)
$f$	function defined in Eq. (9)
$H$	vertical height of an embankment
$H_c$	critical height of an embankment
$L$	length AB in Fig. 1
$N_s$	stability factor
$r_o, r(\theta)$	length variables of a log spiral curve
$V(\theta)$	discontinuous velocity across the failure plane
$\alpha, \beta$	angular parameters of an embankment
$\gamma$	unit weight of soil
$\Omega$	angular velocity
$\phi$	friction angle of soil
$\theta_h, \theta_o$	angular variables of a log spiral curve

## 8. REFERENCES

- Chen, W. F. and C. R. Scawthorn (1968). Limit analysis and limit equilibrium solutions in soil mechanics, Fritz Engineering Laboratory Report No. 355.3, Lehigh University, Bethlehem, Pa.
- Chen, W. F. (1968). Discussion of application of limit plasticity in soil mechanics by W. D. Liam Finn, American Society of Civil Engineers, Journal of the Soil Mechanics and Foundations Division, Vol. 94, No. SM2, American Society of Civil Engineers.
- Chen, W. F. (1969). Soil mechanics and the theorems of limit analysis, Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 95, No. SM2.
- Cox, A. D., G. Eason and H. G. Hopkins (1961). Axially symmetric plastic deformation in soils, transaction, Royal Society of London, Vol. 254 A, p. 1.
- Drucker, D. C., W. Prager and H. J. Greenberg (1952). Extended limit design theorems for continuous media, Quarterly of Applied Mathematics, Vol. 9, pp. 381-389.
- Drucker, D. C. and W. Prager (1952). Soil mechanics and plastic analysis or limit design, Quarterly of Applied Mathematics, Vol. 10, pp. 157-165.
- Drucker, D. C. (1953). Limit analysis of two and three dimensional soil mechanics problems, Journal of the Mechanics and Physics of Solids (Oxford), Vol. 1, No. 4, pp. 217-226.
- Drucker, D. C., R. E. Gibson and D. J. Henkel (1957). Soil mechanics and work-hardening theories of plasticity, Proceedings, American Society of Civil Engineers, Vol. 81, Separate 798, 1955; Transactions; Vol. 122, pp. 338-346.
- Fellenius, W. (1927). Erdstatische Berechnungen. Berlin: W. Ernst und Sohn.
- Finn, Liam W. D. (1967). Application of limit plasticity in soil mechanics, Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, Vol. 93, No. SM5, September, 1967, pp. 101-120.
- Jenike, A. W. and R. T. Shield (1959). On the plastic flow of coulomb solids beyond original failure, Journal of Applied Mechanics, Vol. 26, pp. 599-602.
- Shield, R. T. (1954). Mixed boundary value problems in soil mechanics, Quarterly of Applied Mathematics, Vol. 11, No. 1, pp. 61-76



Shield, R. T. (1955). On Coulomb's law of failure in soils, Journal of the Mechanics and Physics of Solids, Vol. 4, No. 1, pp. 10-16.

Spencer, A. T. M. (1964). A theory of the kinematics of ideal soils under plane strain conditions, Journal of Mechanics and Physics of Solids, Vol. 12, pp. 337-351.

Terzaghi, K. (1943). Theoretical Soil Mechanics. New York: John Wiley and Sons.

Taylor, D. W. (1948). Fundamentals of Soil Mechanics. New York: John Wiley and Sons

Weidler, T. B. and Paslay (1966). An analytical description of the behavior of granular media, Engineering, Brown University Report 36, Arpa SD-86.

## PUBLICATIONS

Geotechnical Engineering Division  
Fritz Engineering Laboratory  
Department of Civil Engineering  
Lehigh University

1. Varrin, R. D. and Fang, H. Y.  
DESIGN AND CONSTRUCTION OF HORIZONTAL VISCOUS  
FLUID MODEL, Ground Water, Vol. 5, No. 3, July  
1967, (Fritz Laboratory Reprint No. 67-10)
2. Fang, H. Y. and Schaub, J. H.  
ANALYSIS OF THE ELASTIC BEHAVIOR OF FLEXIBLE PAVEMENT,  
Proc. 2nd International Conference on the Structural  
Design of Asphalt Pavements, pp. 719-729, August 1967,  
(Fritz Laboratory Reprint No. 67-10)
3. Chen, W. F.  
DISCUSSION OF APPLICATION OF LIMIT PLASTICITY IN  
SOIL MECHANICS by W. D. Liam Finn, ASCE Proc., Journal  
of Soil Mechanics and Foundation Division, Vol. 94,  
SM2, March 1968, (Fritz Laboratory Reprint No. 68-7)
- \*4. Krugmann, P., Boschuk, J., Jr. and Fang, H. Y.  
ANNOTATED BIBLIOGRAPHY ON STEEL SHEET PILE STRUCTURES,  
March 1968, (Fritz Laboratory Report No. 342.1)
5. Drucker, D. C. and Chen, W. F.  
ON THE USE OF SIMPLE DISCONTINUOUS FIELDS TO BOUND  
LIMIT LOADS, "Engineering Plasticity" edited by  
J. Heyman and F. A. Leckie, Cambridge University Press,  
pp. 129-145, March 1968 (Fritz Laboratory Reprint No.  
68-8)
- \*6. Chen, W. F. and Scawthorn, C.  
LIMIT ANALYSIS AND LIMIT EQUILIBRIUM SOLUTIONS IN SOIL  
MECHANICS, June 1968, (Fritz Laboratory Report No. 355.3)
7. Fang, H. Y. and Brewer, C. E.  
FIELD STUDY OF AN ANCHORED SHEET PILE BULKHEAD, ASCE  
Annual Meeting, Pittsburgh, Pa., October 1968, (ASCE  
Preprint No. 749)
- \*8. Chen, W. F.  
EXTENSIBILITY OF CONCRETE OR ROCK AND THE THEOREMS OF  
LIMIT ANALYSIS, November 1968, (Fritz Laboratory Report  
No. 356.5)
- \*9. Moore, C. A. and Hirst, T. J.  
A LOW FRICTION SEAL FOR THE TRIAXIAL TEST LOADING  
PISTON, December 1968, (Fritz Laboratory Report No.  
348.3)
- \*10. Hirst, T. J. and Mitchell, J. K.  
THE INFLUENCE OF SUSTAINED LOAD ON SOIL STRENGTH,  
December 1968, (Fritz Laboratory Report No. 348.4)

---

\*Not for general distribution

11. Chen, W. F.  
ON THE RATE OF DISSIPATION OF ENERGY OF SOILS,  
January 1969, Soils and Foundations, The Japanese  
Society of Soil Mechanics and Foundation Engineering,  
Vol. VIII, No. 4, pp. 48-51, December 1968, (Fritz  
Laboratory Reprint No. 68-24)
- \*12. Dismuke, T. D.  
FIELD STUDY OF SURCHARGE EFFECTS ON A STEEL SHEET  
PILE BULKHEAD, December 1968, (Fritz Laboratory  
Report No. 342.4)
- \*13. Fang, H. Y. and Varrin, R. D.  
MODEL ANALYSIS OF HYDRAULIC CONDUCTIVITY OF AN  
ACQUIFER, December 1968, (Fritz Laboratory Report  
No. 341.2)
14. Fang, H. Y.  
INFLUENCE OF TEMPERATURE AND OTHER CLIMATIC FACTORS  
ON PERFORMANCE OF SOIL-PAVEMENT SYSTEMS, International  
Symposium on the Effects of Temperature and Heat on  
the Engineering Behavior of Soils, January 1969,  
(Fritz Laboratory Reprint No. 69-2)
- \*15. Brewer, C. E. and Fang, H. Y.  
FIELD STUDY OF SHEAR TRANSFER IN STEEL SHEET PILING,  
February 1969, (Fritz Laboratory Report No. 342.3)
16. Chen, W. F.  
SOIL MECHANICS AND THE THEOREM OF LIMIT ANALYSIS,  
ASCE Proc., Journal of Soil Mechanics and Foundation  
Division, Vol. 95, No. SM2, March 1969, (Fritz  
Laboratory Reprint No. 69-4)
- \*17. Chen, W. F., Giger, M. and Fang, H. Y.  
ON THE LIMIT ANALYSIS OF STABILITY OF SLOPES, March 1969,  
(Fritz Laboratory Report No. 355.4)
18. Corman, R. E., Jr. and Miller, F. H.  
MODIFIED RUT DEPTH GAUGE, Envirotronics Bulletin No. 2,  
March 1969. (available from Envirotronics Corporation,  
P. O. Box 594, Bethlehem, Pa. 18015)
19. Chen, W. F. and Drucker, D. C.  
THE BEARING CAPACITY OF CONCRETE BLOCKS OR ROCK, ASCE  
Proc., Journal of Engineering Mechanics Division, Vol. 95,  
No. EM2, April 1969, (Fritz Laboratory Reprint No. 69-1)
20. Chen, W. F.  
THE PLASTIC INDENTATION OF METAL BLOCKS BY A FLAT PUNCH,  
October 1968, to be presented at the Plasticity Session  
of the Louisville, Ky. ASCE National Structural Confer-  
ence, April 14-18, 1969